PARTICULAR SOLUTIONS OF UNSTEADY MHD EQUATIONS WITH ACCOUNT FOR THERMAL CONDUCTIVITY FOR FINITE MAGNETIC REYNOLDS NUMBERS

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We examine unsteady plane-symmetric and axisymmetric motions of a thermally conductive gas of finite conductivity in a magnetic field normal to the direction of motion of the medium.

We seek a solution with velocity field having a linear dependence on the space coordinate. The resulting system of equations is solved by separation of variables.

Some numerical calculations are presented for the problem of plane-symmetric expansion of a conducting gas in a magnetic field in the case in which the electric and thermal conductivities depend only on the temperature. Distributions of the temperature, density, and magnetic field across the layer section are presented as a function of magnetic Reynolds number and dimensionless thermal conductivity.

We examine one-dimensional plane-symmetric and axisymmetric expansions of a conducting gas in a magnetic field directed along the Z axis (perpendicular to the direction of gas motion). At the initial time $t = 0$ the gas occupies the space $-a_0 \le x \le a_0$ between two planes (in the plane-symmetric case) or is a cylindrical column of radius a_0, infinite along the Z axis.

The following assumptions are made.

1) The gas satisfies the ideal gas equation of state and the viscosity is zero.

2) The displacement currents are negligibly small. The magnetic field at the outer edge of the conducting gas can be specified without examining the in vacuo wave processes (quasistationary electromagnetic field).

2) The electric conductivity σ_1 and thermal conductivity λ_1 of the gas depend on the temperature and density following power-law relations

$$\sigma_1 = \sigma_0 \left(\frac{T}{T_0}\right)^n \left(\frac{\rho}{\rho_0}\right)^r, \qquad \lambda_1 = \lambda_0 \left(\frac{T}{T_0}\right)^m \left(\frac{\rho}{\rho_0}\right)^k \tag{1}$$

4) We examine uniform expansion with the velocity $v_x = xa'(t)/a(t)$, which depends linearly on the space coordinate. Here $a(t)$ is the unknown law of motion of the gas boundary.

Under these assumptions the dimensionless MHD equations in Lagrangian coordinates (ξ, τ) have the form

$$\frac{\partial}{\partial\tau}[\mu^{\gamma+1}\rho(\xi,\tau)] = 0, \qquad \varkappa\xi\rho(\xi,\tau)\mu\mu'' = -\frac{\partial}{\partial\xi}[p(\xi,\tau) + h_1^2(\xi,\tau)]$$
$$\frac{\partial}{\partial\tau}[\mu^{\gamma+1}(\tau)h_1(\xi,\tau)] = \frac{1}{R_m\xi^\gamma\mu^2}\frac{\partial}{\partial\xi}\left[\frac{\xi^\gamma}{\Theta_1^n\rho^r}\frac{\partial}{\partial\xi}(\mu^{\gamma+1}h_1)\right], \qquad p = \rho\Theta_1$$
$$\frac{\partial\Theta_1}{\partial\tau} = -(\varkappa-1)p\frac{\partial}{\partial\tau}\left(\frac{1}{\rho}\right) + \frac{2(x-1)}{R_m}\frac{1}{\Theta_1^n\rho^{r+1}}\frac{1}{\mu^2}\left(\frac{\partial h_1}{\partial\xi}\right)^2 + \frac{\Omega}{\xi^\gamma\mu^2\rho}\frac{\partial}{\partial\xi}\left[\Theta_1^m\rho^k\xi^\gamma\frac{\partial\Theta_1}{\partial\xi}\right] \tag{2}$$
$$\xi = \frac{x}{a(t)}, \quad \tau = \frac{t}{t_0}, \quad \mu = \frac{a(t)}{a_0}, \quad h_1 = \frac{H}{H_0}, \quad \Theta_1 = \frac{T}{T_0}, \quad \varkappa = \frac{c_p}{c_v}, \quad t_0 = \frac{a_0}{v_0}, \quad R_m = \frac{4\pi\sigma_0 v_0 a_0}{c^2}$$

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Here R_m is the magnetic Reynolds number

$$\Omega = \frac{8\pi\lambda_0 t_0 RT_0}{a_0^2 H_0^2 c_v} = \frac{8\pi(\varkappa - 1)\lambda_0 T_0}{v_0 a_0 H_0^2}$$

The dimensionless pressure p and density ρ are obtained by referring the dimensional quantities to the scales

$$p_0 = \frac{H_0^2}{8\pi}, \qquad \rho_0 = \frac{p_0}{RT_0} = \frac{H_0^2}{8\pi RT_0}$$

We take as the H_0, T_0 scales the magnetic field intensity and gas temperature at the outer boundary at the time t = 0, the characteristic velocity v_0 is the speed of sound $\sqrt{\varkappa RT_0}$ corresponding to the temperature T_0.

In the plane-symmetric case $\gamma = 0$, in the cylindrical case $\gamma = 1$. From (2.1)

$$\rho(\xi, \tau) = \frac{\Phi(\xi)}{\mu^{\gamma+1}(\tau)} \tag{3}$$

After substituting (3) and converting to the new functions

$$h(\xi, \tau) = \mu^{\gamma+1} h_1(\xi, \tau), \qquad \theta(\xi, \tau) = \mu^{(\gamma+1)(\varkappa-1)}\theta_1(\xi, \tau) \tag{4}$$

the remaining equations (2) take the form

$$\varkappa\xi\Phi(\xi)\frac{\mu''}{\mu^\gamma} = -\frac{\partial}{\partial\xi}\left[p(\xi, \tau) + \frac{h^2(\xi, \tau)}{\mu^{2(\gamma+1)}}\right], \quad \frac{\partial h}{\partial\tau} = \frac{\mu^{(\gamma+1)[n(\varkappa-1)+r]-2}}{R_m\xi^\gamma}\frac{\partial}{\partial\xi}\left[\frac{\xi^\gamma}{\theta^n\Phi^r}\frac{\partial h}{\partial\xi}\right]$$

$$\frac{\partial\theta}{\partial\tau} = \frac{2(\varkappa-1)}{R_m}\frac{\mu^{(\gamma+1)[r+\varkappa+n(\varkappa-1)-2]-2}}{\theta^n\Phi^{r+1}}\left(\frac{\partial h}{\partial\xi}\right)^2 + \Omega\frac{\mu^{(\gamma+1)[1-k-m(\varkappa-1)]-2}}{\xi^\gamma\Phi(\xi)}\frac{\partial}{\partial\xi}\left[\theta^m\Phi^k\xi^\gamma\frac{\partial\theta}{\partial\xi}\right] \tag{5}$$

$$p = \frac{\Phi(\xi)}{\mu^{\varkappa(\gamma+1)}}\theta(\xi, \tau)$$

We seek the particular solution of (5) by separation of variables, analogous to [1].

In [2] this approach was used to obtain the solution of the subject problem without account for thermal conductivity. This solution was characterized by the requirement for existence of an external backpressure proportional to the magnetic pressure. The solution without backpressure, i.e.,expansion into a vacuum, could not exist because of finite Joule heat release at the outer boundary (the conductivity depended only on the temperature) for zero density of the medium, therefore in the absence of thermal conductivity the temperature could not be bounded at the outer boundary. Account for thermal conductivity makes it possible to obtain the solution for the case of expansion into a vacuum as well.

Let

$$h(\xi, \tau) = G(\tau) Z(\xi), \qquad \theta(\xi, \tau) = V(\tau) X(\xi)$$

Then from (5.4)

$$p(\xi, \tau) = \Phi(\xi) X(\xi)\frac{V(\tau)}{\mu^{\varkappa(\gamma+1)}(\tau)} \tag{6}$$

Substitution of (6) into (5) yields

$$\frac{\mu^{\varkappa(\gamma+1)-\gamma}\mu''}{V(\tau)} = -\frac{1}{\varkappa\xi\Phi(\xi)}\left[(\Phi X)' + \frac{G^2(\tau)\mu^{(\gamma+1)(\varkappa-2)}}{V(\tau)}(Z^2)'\right], \quad \frac{G'(\tau)}{G(\tau)}\frac{V^n(\tau)}{\mu^{(\gamma+1)[r+n(\varkappa-1)]-2}} = \frac{1}{R_m\xi^\gamma Z}\left[\frac{\xi^\gamma}{X^n\Phi^r}Z'\right]'$$

$$\frac{V'(\tau)}{V^{(m+1)}(\tau)}\mu^{2-(\gamma+1)[1-k-m(\varkappa-1)]} = \frac{2(\varkappa-1)}{R_m}\frac{G^2(\tau)\mu^{(\gamma+1)[(n+m)(\varkappa-1)+r+k+\varkappa-3]}}{V^{n+m+1}(\tau)} \times \frac{(Z')^2}{X^{n+1}\Phi^{r+1}} + \Omega\frac{(X^m\Phi^k\xi^\gamma X')'}{\xi^\gamma\Phi X} \tag{7}$$

Here the left sides of the equations depend only on the variable τ, the right sides of (7.1) and (7.3) contain mixed terms. The functions $\mu(\tau)$, $G(\tau)$, $V(\tau)$, $\Phi(\xi)$, $X(\xi)$, $Z(\xi)$ taken together will satisfy (7) if the functions of τ and the functions of ξ individually satisfy the following systems of equations

$$\mu^{\varkappa(\gamma+1)-\gamma}(\tau)V^{-1}(\tau)\mu'' = N_1, \qquad G^2(\tau)V^{-1}(\tau)\mu^{(\gamma+1)(\varkappa-2)} = 1$$
$$G^{-1}(\tau)V^n(\tau)\mu^{2-(\gamma+1)[r+n(\varkappa-1)]}G'(\tau) = N_2, \quad V^{-(m+1)}(\tau)\mu^{2-(\gamma+1)[1-k-m(\varkappa-1)]}V'(\tau) = N_3 \tag{8}$$
$$G^2(\tau)V^{-(n+m+1)}\mu^{(\gamma+1)[(n+m)(\varkappa-1)+r+k+\varkappa-3]} = 1, \quad (\Phi X)' + (Z^2)' = -N_1\varkappa\xi\Phi(\xi)$$

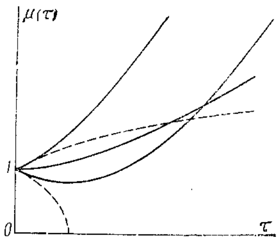

Fig. 1

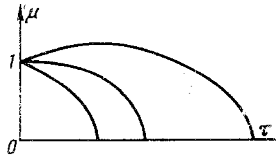

Fig. 2

$$\frac{1}{R_m \xi^\gamma Z}\left[\frac{\xi^\gamma}{X^n \Phi^r} Z'\right] = N_2, \quad \frac{2(\varkappa - 1)}{R_m} \frac{(Z')^2}{X^{n+1}\Phi^{r+1}} + \Omega \frac{(X^m \Phi^k \xi^\gamma X')'}{\xi^\gamma \Phi X} = N_3 \tag{9}$$

Here N_1, N_2, N_3 are constants; on the basis of (10) the constants in (8.2) and (9.5) equal unity. In (9) the number of equations is equal to the number of unknowns, the system (8) contains five equations for the three unknowns $\mu(\tau)$, $G(\tau)$, $V(\tau)$; these equations are compatible only for certain limitations imposed on the constants N_1, N_2, N_3, and the physical parameters $\varkappa$, n, m, r, k. We note that (8) and (9) are not the only possible systems. Specifically, from (7) we can obtain a system of five equations for the three functions of ξ; in this case a system of three equations is obtained for the function of τ. However, study of the resulting equations showed that their compatibility is possible for limitations which make the solutions trivial (for example, the solution with zero magnetic field gradient).

The functions $\mu(\tau)$, $G(\tau)$, $V(\tau)$ must satisfy the initial conditions

$$\mu(0) = 1, \qquad \mu'(0) = \pm M_0, \qquad G(0) = 1$$
$$V(0) = 1 \left(M_0 = \frac{1}{\sqrt{\varkappa R T_0}} \left| \frac{da}{dt} \right|_{t=0}\right) \tag{10}$$

Here M_0 is the Mach number for $t = 0$.

The functions $\Phi(\xi)$, $X(\xi)$, $Z(\xi)$ must satisfy the boundary conditions

$$Z'(0) = 0, \quad \Phi(1) = 0, \quad X(1) = 1, \quad Z(1) = 1, \quad X^m \Phi^k X' |_{\xi=1} = 0 \tag{11}$$

The conditions (11.2) and (11.5) correspond to the problem of expansion or contraction of a layer in vacuum, when the density of the medium and the thermal flux at the outer boundary equal zero (radiation is not considered).

Turning to the study of (8), we must examine several cases separately.

<u>A. Case $N_3 = N_2 = 0$.</u> This case corresponds to $\partial h/\partial \tau = 0$, $\partial \theta/\partial \tau = 0$ in (5). In this case, naturally,

$$\frac{\partial h_1}{\partial \tau} \neq 0, \qquad \frac{\partial \Theta_1}{\partial \tau} \neq 0$$

The compatibility conditions for (8) with $N_2 = N_3 = 0$ impose two limitations on the physical constants $\varkappa$, n, m, r, k

$$\varkappa = 2, \qquad n + m + r + k - 1 = 0 \tag{12}$$

The constant N_1 is arbitrary. For the functions $G(\tau)$ and $V(\tau)$ we obtain the solutions

$$G(\tau) \equiv 1, \qquad V(\tau) \equiv 1 \tag{13}$$

For $\mu(\tau)$ we obtain the second-order differential equation $\mu^{2+\gamma}\mu'' = N_1$, which can be integrated once and with account for the conditions (10) can be written in the form

$$\left(\frac{d\mu}{d\tau}\right)^2 = M_0^2 + \frac{2N_1}{1+\gamma}\left(1 - \frac{1}{\mu^{1+\gamma}}\right) \tag{14}$$

The nature of the solution $\mu(\tau)$ as a function of the constant N_1 and the initial velocity $\mu'(0)$ can be investigated immediately from this equation.

If $N_1 > 0$, $\mu'(0) = M_0 > 0$, then $\mu(\tau)$ increases monotonically, and

$$\mu'(\tau) \to M^* \quad \text{as} \quad \tau \to \infty$$
$$(M^* = \sqrt{M_0^2 + 2N_1/(1+\gamma)}) \tag{15}$$

For $N_1 > 0$, $\mu(0) = -M_0 < 0$ the function $\mu(\tau)$ initially decreases; upon reaching the value $\mu_{\min}$

$$\left(0 < \mu_{\min}^{1+\gamma} = \frac{2N_1}{(1+\gamma)M_0^2 + 2N_1} < 1\right)$$

the velocity $\mu'(\tau)$ becomes equal to zero and thereafter $\mu(\tau)$ begins to increase monotonically, and the velocity $\mu'(\tau)$ approaches asymptotically the same magnitude $\mu'_{\max} = M^*$ as in the case $N_1 > 0$, $\mu'(0) = M_0$.

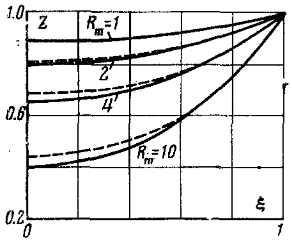

Fig. 3

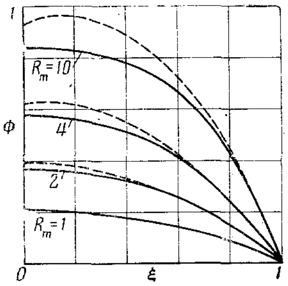

Fig. 4

Let $N_1 < 0$ and the quantity $M_0^2 + 2N_1/(1+\gamma) < 0$. If in this case $\mu'(0) = M_0 > 0$, $\mu(\tau)$ is a monotonically increasing function and $\mu'(\tau) \to M^* < M_0$ as $\tau \to \infty$. However, if $\mu'(0) = -M_0 < 0$, $\mu(\tau)$ decreases monotonically and reaches the value $\mu = 0$ for which $\mu'(\tau) = -\infty$. For more clarity the approximate nature of the curves $\mu(\tau)$ for the cases being considered is shown in Fig. 1. Here the relations for $N_1 < 0$ are shown dashed, those for the case $N_1 > 0$ are shown by the continuous curves. The nature of the curves for the case $N_1 < 0$, $M_0^2 + 2N_1/(1+\gamma) < 0$ is shown in Fig. 2.

Hence we see that for $\mu'(0) = M_0 > 0$ the quantity $\mu(\tau)$ first increases, reaches the maximal value μ_{max}, then falls to zero, and

$$\mu_{max} = \frac{2N_1}{(1+\gamma)[M_0^2 + 2N_1/(1+\gamma)]} > 1$$

For the case of motion with axial symmetry, i.e, for $\gamma = 1$, (14) can be integrated. We obtain

$$\mu(\tau) = \left(\frac{N_1 + [M_0 \pm (M_0^2 + N_1)\tau]^2}{M_0^2 + N_1}\right)^{1/2}$$

Here the plus sign corresponds to a positive initial velocity $\mu'(0) = M_0 > 0$, the minus sign corresponds to the case $\mu'(0) = -M_0 < 0$.

Thus we see that the nature of the motion of the conducting plasma boundary varies over quite wide limits, depending on the arbitrary constants N_1 and the magnitude of the initial velocity $\mu'(0) = \pm M_0$. It is easy to see from (3), (4), (6), and (13) that the time variation of the density, temperature, and magnetic field intensity are determined by the function $\mu(\tau)$; with increase of $\mu(\tau)$ all these parameters decrease, with decrease of $\mu(\tau)$ they increase. Specifically, the region of collapse of the original slug to zero dimensions corresponds to increase of the external magnetic field to infinity; in this case the temperature and density of the medium also increase to infinity. We note that the Mach number, based on the boundary velocity and temperature as follows

$$M = \frac{da/dt}{\sqrt{\varkappa RT(\xi = 1)}} = \frac{\mu'(\tau)}{\sqrt{\Theta_1(1,\tau)}} = \mu'(\tau)\,\mu^{1/2(\gamma+1)}(\tau),$$

approaches infinity with infinite expansion of the slug regardless of the initial value of M_0.

We turn to the equation (9) for the functions of ξ in the considered case (A) $N_2 = N_3 = 0$. In this case the equations (9.2) and (9.3) can be integrated once

$$\frac{\xi^\gamma}{X^n \Phi^r} Z' = C_1, \qquad \frac{2C_1}{R_m} Z + \Omega X^m \Phi^k \xi^\gamma X' = C_2 \tag{16}$$

Here C_1, C_2 are constants of integration.

Further integration of (9.1), (16) for arbitrary parameters n, m, r, k, $\varkappa$, γ, satisfying the conditions (12) is difficult. Below we consider as an example one of the particular cases $n = 1$, $m = k = r = 0$, $\gamma = 0$.

From the equations (9.1), (16) and the boundary conditions (11)

$$X(\xi) = \sin {}^1\!/_2\pi\xi, \qquad Z(\xi) = 1 - \sqrt{{}^1\!/_2 \Omega R_m}\cos {}^1\!/_2\pi\xi$$

$$\Phi(\xi) = \pi\sqrt{{}^1\!/_2\Omega R_m}\,\frac{e^{-2N_1 f(\xi)}}{\sin {}^1\!/_2\pi\xi}\int_\xi^1 [1 - \sqrt{{}^1\!/_2\Omega R_m}\cos {}^1\!/_2\pi\xi]\, e^{2N_1 f(\xi)} d\xi, \quad f(\xi) = \int_1^\xi \frac{\xi\, d\varepsilon}{\sin {}^1\!/_2\pi\xi} \tag{17}$$

We see from (17) that for $\sqrt{{}^1\!/_2\Omega R_m} > 1$ the magnetic field near the plane of symmetry has a direction opposite that of the external field. The density in the plane of symmetry is infinite and $\Phi(\xi) > 0$ only for values of $\sqrt{{}^1\!/_2\Omega R_m}$, not exceeding the critical value. For $N_1 = 0$, i.e., for motion with constant velocity, we see easily from (17.3) that the critical value equals two.

This means that the resulting solution is physically meaningful only for $0 < \sqrt{{}^1\!/_2\Omega R_m} < 2$. We also note that this solution is characterized by the presence of heat removal in the plane of symmetry, whose intensity is defined by the quantity

$$\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = \lambda_0 \frac{T_0}{a_0} \frac{1}{\mu(\tau)} \frac{\partial \Theta_1}{\partial \xi}\bigg|_{\xi=0} = \pi\lambda_0 \frac{T_0}{a_0} \frac{1}{\mu^2(\tau)}$$

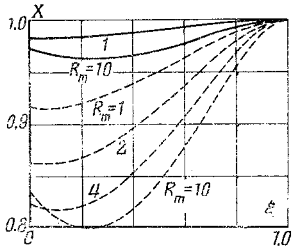

Fig. 5

B. Case $N_2 \neq 0$, $N_3 \neq 0$, $n + m \neq 0$. In this case the simultaneous solution of (8), satisfying the conditions (10), has the form

$$\mu(\tau) = [1 + \mu'(0) s\tau]^{1/s} \quad \left(s = 2 + \frac{(\gamma+1)[n(k-1) - mr]}{n+m}\right)$$

$$G(\tau) = [\mu(\tau)]^{(\gamma+1)(n+m+r+k-1)/2(n+m)}, \quad V(\tau) = [\mu(\tau)]^{(\gamma+1)[(n+m)(\varkappa-1)+r+k-1]/(n-m)} \qquad (18)$$

For compatibility of the system (8) the physical constants n, m, k, r, $\varkappa$, γ, must satisfy the relation

$$(\gamma + 1)\{r + k - 1 + 2\,[n(k-1) - mr]\} + 2\,(n + m) = 0 \qquad (19)$$

The constants N_1, N_2, N_3 are defined uniquely in terms of n, m, k, r, $\varkappa$, γ, μ' (0)

$$N_1 = -\left[1 + (\gamma+1)\frac{n(k-1) - mr}{n+m}\right][\mu'(0)]^2$$

$$N_2 = \frac{(\gamma+1)(n+m+r+k-1)}{2(n+m)}\,\mu'(0)$$

$$N_3 = \frac{(\gamma+1)[(n+m)(\varkappa-1)+r+k-1]}{n+m}\,\mu'(0) \qquad (20)$$

We note that if $N_1 = 0$, (20.1) yields the connection between the physical constants n, m, k, r, γ; in this case the need for the condition (19) disappears.

The system of equations (9) with the conditions (11) can be integrated numerically on a computer. The numerical results presented below were obtained for the plane symmetric case ($\gamma = 0$) for k = r = 0, m = $^1/_2$, n = $^3/_2$ (i.e., $\sigma \sim T^{3/2}$, $\lambda \sim T^{1/2}$), $\varkappa = {}^5/_3$ [these constants satisfy the condition (19)]. The magnetic Reynolds number R_m varies from 1 to 10, the parameter Ω characterizing the thermal conductivity takes the values 0.1 and 0.5.

Figures 3, 4, 5 show the spatial distributions respectively of the magnetic field intensity, gas density, and temperature for $M_0 = 0.5$ for different R_m and two values of Ω. The dashed curves are for $\Omega = 0.1$, continuous curves are for $\Omega = 0.5$. For $R_m = 1$ the functions $Z(\xi)$, $\Phi(\xi)$ for $\Omega = 0.1$ and $\Omega = 0.5$ practically coincide, therefore they are shown by a single curve in Figs. 3 and 4. We see from Fig. 5 that for R_m = 10 there is a temperature gradient in the plane of symmetry, and the thermal flux is directed from the plane of symmetry. Thus, the solution obtained is characterized by the presence of a planar heat source. For smaller values of R_m the temperature gradient in the plane of symmetry becomes so small that it cannot be shown in the figures.

Calculations with $R_m = 2$, $\Omega = 0.5$ for values of M_0 from 0.1 to 2 were made to clarify the influence of M_0 on the functions $Z(\xi)$, $\Phi(\xi)$, $X(\xi)$. All the curves differed very little from those obtained for $M_0 = 0.5$, therefore they are not presented in graphical form.

It is not difficult to note that for all the parameter values examined the temperature had a maximum at the outer boundary; the same sort of temperature distribution occurs in the absence of thermal conductivity [2]. The occurrence of a high-temperature layer (T-layer) was discovered in [3], and the T-layer is not necessarily located at the outer boundary of the expanding slug. The particular solutions obtained in the present study also permit having the maximal temperature within the layer (i.e., for $\xi < 1$); for this to occur, in place of the condition (11.5) for absence of thermal flux at the outer boundary it is necessary to examine the condition $X^m \Phi' X'_{\xi=1} = q < 0$, which accounts for the possibility of heat transfer from the outer boundary (radiation). This is most readily seen in case (A) from the equations (9.1) and (16).

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